Chapter Ten

The Straight Line

The gradient:

The gradient of a line which passes through the points (x_1,y_1) and (x_2, y_2) is given by $\frac{y_2 - y_1}{x_2 - x_1}$.
The gradient is also called the slope.

Q1. Find the gradient of the line passing through the points (2, 5) and (7, 4).

Solution

Let (x_1 , y_1) = (2, 5)=>x_1 = 2 and y_1 = 5.
Also let (x_2, y_2) = (7 4) => x_2, = 7 and y_2 = 4.
The gradient $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{4 - 5}{7 - 2} = \frac{-1}{5} = -0.2$.

Q2. Find the slope or the gradient of the line which joins the points (-5, 2) and (8, -4).

Solution

Let (x_1 , y_1) = (-5, 2) =>x_1 = -5 and y_1, = 2.
Also let (x_2, y_2) = (8,- 4)
=> x_2 = 8 and y_2 = - 4.
The slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{8 - (-5)} = \frac{-6}{8+5} = -\frac{0.46}{13}$.

Q3. Determine the slope of the line which joins the points (- 2, - 4) and (- 6, - 8).

Solution

Let (x_1, y_1) = (-2, - 4) => x_1 = -2 and y_1, = - 4.
Also let (x_2, y_2) = (- 6, - 8) => x_2 = - 6 and y_2 = - 8.
The gradient $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{-8 - (-4)}{-6 - (-2)} = -\frac{8 + 4}{-6 + 2} = -\frac{4}{-4} = 1$.

Q4. The gradient of the line which passes through the points (2,4) and (6,Q) is 2. Find the value of Q.

Solution

Let (x_1, y_1) = (2,4) => x_1 = 2 and y_1= 4.
Also let (x_2, y_2) = (6,Q) => x_2 = 6 and y_2 = Q.
The gradient = $\frac{y_2 - y_1,}{x_2 - x_1,} = \frac{Q - 4}{6 - 2}$
Since the graduate = 2, then $2 = \frac{Q - 4}{6 - 2}$ => $2 = \frac{Q - 4}{4}$
=>2 x 4 = Q – 4 => 8 = Q - 4
=> 8 + 4 = Q => Q = 12.

Q5. If the slope of the line which joins the points (1, 6) and (m, - 4) is 5, determine the value of m.

Solution

Let $(x_1, y_1) = (1, 6) \Rightarrow x_1 = 1$ and $y_1 = 6$.
Also let $(x_2, y_2) = (m, -4) \Rightarrow x_2 = m$ and $y_2 = -4$.

Slope $= \frac{y_2 - y_1,}{x_2 - x_1,} = -\frac{4 - 6}{m - 1}$

Since the slope = 5, then $5 = -\frac{4 - 6}{m - 1}$

$\Rightarrow 5(m - 1) = -10$

$\Rightarrow 5m - 5 = -10 \Rightarrow 5m = -10 + 5,$
$\Rightarrow 5m = -5 \Rightarrow m = \frac{-5}{5} = -1.$

Q6.Determine the gradient of the line which joins the points $(^1/_2, ^1/_4)$ and $(^2/_3, ^3/_5)$.

Solution

Since ½ = 0.2, ¼ = 0.25, $^2/_3 = 0.66$ and $^3/_5 = 0.6$, then
the line joins the points (0.2, 0.25) and (0.66, 0.6).
Let $(x_1, y_1) = (0.2, 0.25) \Rightarrow x_1 = 0.2$ and $y_1 = 0.25$.
Also let $(x_2, y_2) = (0.66, 0.6)$,
$\Rightarrow x_2 = 0.66$ and $y_2 = 0.6$.

The gradient $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{0.6 - 0.25}{0.66 - 0.2} = \frac{0.35}{0.46} = 0.8.$

Q7. Determine the slope of the line joining the points $(^{-4}/_5, ^{-1}/_3)$ and $(^3/_7, ^{-1}/_6)$.

Solution

$^{-4}/_5 = -0.8$, $^{-1}/_3 = -0.33$, $^3/_7 = 0.4$ and $^{-1}/_6 = -0.17$.
The line therefore joins the points (- 0.8, - 0.33) and
(0.4, - 0.17).
Let $(x_1, y_1) = (-0.8, -0.33) \Rightarrow x_1 = -0.8$ and $y_1 = -0.33$.
Also let $(x_2, y_2) = (0.4, -0.17) \Rightarrow x_2 = 0.4$ and $y_2 = -0.17$.

Slope $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{-0.17 - (-0.33)}{0.4 - (-0.8)} = -\frac{0.17 + 0.33}{0.4 + 0.8}$

$= \frac{0.16}{1.2} = 0.13$

Types of equations of straight line:

There are two types of equations of the straight line
and these are (1) y = mx (2) y = mx + c

Lines with equation of the form y = mx:

- In the equation y = mx, m = the slope or the gradient.
- All such lines pass through the origin or the point (0, 0).

- Examples are lines with equations:
(a) y = 2x, in which the slope is 2.
(b) y = 5x in which the slope is 5.
(c) y = -3x in which the gradient = -3.

Lines with equation of the form y = mx + c:

- In y = mx + c, m = the gradient and c = the y intercept i.e. the point where the graph or line cuts the y-axis.
- Examples of such line graphs are
(1) y = 2x +5, in which the slope is 2 and the y intercept is 5.
(2) y = -3x + 4, in which the slope =-3 and the y intercept = 4.

The equation $y - y_1 = m(x - x_1)$:

- The equation of the straight line with slope m and which passes through the point (x_1, y_1), is given by $y - y_1 = m(x - x_1)$.
- This equation is used when the slope of the line as well as one point through which the line passes are given.

Q1. Find the equation of the line which passes through the point (4, 2) and whose slope is 5.

Solution

Since the slope is 5 => m = 5.
Let $(x_1, y_1) = (4, 2) \Rightarrow x_1 = 4$ and $y_1 = 2$.
Using $y - y_1 = m(x - x_1)$
$\Rightarrow y - 2 = 5(x - 4)$,
$\Rightarrow y - 2 = 5x - 20 \Rightarrow y = 5x - 20 + 2$,
$\Rightarrow y = 5x - 18$.

Q2 Determine the equation of the line whose slope is -3, if it passes through the point (- 4, 8).

Solution

The slope = -3 => m = -3.
Let $(x_1, y_1) = (-4, 8) \Rightarrow x_1 = -4$ and $y_1 = 8$.
Since $y - y_1 = m(x - x_1)$, then $y - 8 = -3\{x - (-4)\}$,
$\Rightarrow y - 8 = -3\{x + 4\}$
$\Rightarrow y - 8 = -3x - 12$,
$\Rightarrow y = -3x - 12 + 8$
$\Rightarrow y = -3x - 4$.

Q3. A line which has a gradient of $^{-1}/_{3}$ passes through the point (- 6, -12). Determine its equation.

Solution

The slope of the line = $^{-1}/_{3}$
$\Rightarrow m = {}^{-1}/_{3} = -0.33$.
Let $(x_1, y_1) = (-6, -12)$
$\Rightarrow x_1 = -6$ and $y_1 = -12$.
But $y - y_1 = m(x - x_1)$
$\Rightarrow y - (-12) = -0.33(x - (-6))$,
$\Rightarrow y + 12 = -0.33(x + 6)$

=> y + 12 = - 0.33x – 1.98,
=> y = - 0.33x – 1.98 -12
=> y = - 0.33x – 13.98.

Q4. Determine the equation of the line whose slope is $^{-1}/_{2}$, if it passes through the point ($^{-3}/_{4}$, $^{-2}/_{5}$).

Solution

Since the slope = $^{-1}/_{2}$ => m = $^{-1}/_{2}$ = - 0.5.
$^{-3}/_{4}$ = - 0.75 and $^{-2}/_{5}$ = - 0.4,
=> the line passes through the point (- 0.75, -0.4).
Let (x_1, y_1) = (- 0.75, - 0.4)
=> x_1 = - 0.75 and y_1 = -0.4.
From $y – y_1 = m (x – x_1)$
=>y – (- 0.4) = - 0.5 (x – (-0.75)),
=> y + 0.4 = - 0.5(x + 0.75)
=> y + 0.4 = - 0.5x – 0.38,
=> y = - 0.5x – 0.38 – 0.4
=> y = - 0.5x – 0.78.

Q5. Determine the equation of the line, whose slope is $^{-2}/_{3}$, if it passes through the point ($-1^{1}/_{2}$, $1^{1}/_{3}$).

Solution

Since the slope = $^{-2}/_{3}$ => m = $^{-2}/_{3}$ = - 0.66.
$1^{1}/_{2}$ = $^{-3}/_{2}$ = -1.5 and $1^{1}/_{3}$ = $^{4}/_{3}$ = 1.33,
=> the line passes through the point (-1.5, 1.33).
Let (x_1, y_1) = (-1.5, 1.33) => x_1 = -1.5 and y_1 = 1.33.
From $y – y_1 = m (x – x_1)$
=> y - 1.33 = - 0.66 {x – (-1.5)},
=> y – 1.33 = - 0.66 (x + 1.5)
=> y – 1.33 = - 0.66x – 0.99,
=> y = - 0.66x – 0.99 + 1.33
=> y = - 0.66x + 1.33 – 0.99,
=> y = - 0.66x + 0.34.

The equation $y - y_1 = \frac{y_2 – y_1}{x_2 – x_1} (x – x_1)$

- This formula is used when the gradient is not given, but two points through which the line passes are given.
- Also in this given formula, $\frac{y_2 – y_1}{x_2 – x_1}$ = the slope.

Q1. Find the equation of the line which passes through the points (4, 2) and (5, 3).

Solution

Let (x_1,y_1) = (4,2) => x_1 = 4 and y_1 = 2.
Also let (x_2, y_2) = (5, 3) => x_2 = 5 and y_2 = 3.

But $y – y_1 = \frac{y_2 – y_1}{x_2 – x_1} (x – x_1)$

=> $y – 2 = 3 – 2 \quad (x – 4)$

$\frac{}{5-4}$

$\Rightarrow y - 2 = {}^{1}/_{1}(x - 4)$

$\Rightarrow y - 2 = 1(x - 4)$,

$\Rightarrow y - 2 = x - 4 \Rightarrow y = x - 4 + 2 \Rightarrow y = x - 2$.

Q2. A line joins the points (8, 5) and (- 6, 4). Determine its equation.

Solution

Let $(x_1, y_1) = (8, 5) \Rightarrow x_1 = 8$ and $y_1 = 5$.

Also let $(x_2, y_2) = (- 6, 4) \Rightarrow x_2 = - 6$ and $y_2 = 4$.

Since $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Then $y - 5 = \frac{4 - 5}{-6 - 8}(x - 8)$

$\Rightarrow y - 5 = \frac{-1}{-14}(x - 8)$

$\Rightarrow y - 5 = 0.07(x - 8)$,

$\Rightarrow y - 5 = 0.07x - 0.56$

$\Rightarrow y = 0.07x - 0.56 + 5$,

$\Rightarrow y = 0.07x + 5 - 0.56$

$\Rightarrow y = 0.07x + 4.44$.

Q3. Find the equation of the line joining the points (- 6, - 2) and (- 3, 4).

Solution

Let $(x_1, y_1) = (- 6, -2) \Rightarrow x_1 = - 6$ and $y_1 = -2$.

Also let $(x_2, y_2) = (- 3, 4)$

$\Rightarrow x_2 = -3$ and $y_2 = 4$.

Since $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - (-2) = \frac{4 - (-2)}{-3 - (-6)}\{x - (-6)\}$

$\Rightarrow y + 2 = \frac{4 + 2}{-3 + 6}(x + 6)$,

$\Rightarrow y + 2 = \frac{6}{3}(x + 6)$,

$\Rightarrow y + 2 = 2(x + 6)$

$\Rightarrow y + 2 = 2x + 12$,

$\Rightarrow y = 2x + 12 - 2$

$\Rightarrow y = 2x + 10$.